Fast Solution of Induction Heating Problems by Structure Preserving Nonlinear Model Order Reduction

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A fast approach to the numerical solution of induction heating problems, exploiting a novel nonlinear Model Order Reduction technique, is proposed. The problem equations are rewritten in an equivalent way in which only quadratic nonlinearities occur. Projecting these equations in such a way to preserve their nonlinear structure, a reduced nonlinear model is directly constructed. The projection space is attained by numerically computing a few of the first kernels in the Volterra series expansion of the solution to the induction heating problem. Numerical results show that the construction of the proposed reduced nonlinear model is performed at a computational cost which is about one order of magnitude less than that of a standard approach. The reduced model solution is numerically performed at negligible computational cost and allows to reconstruct the whole space-time distribution of the coupled electromagnetic and thermal field, with high accuracy.

***Index Terms*—Induction heating, Eddy currents, Nonlinear dynamical systems, Model order reduction, Volterra series**

I. Introduction

INDUCTION heating problems are coupled electromagnetic and thermal problems in which nonlinearities occur due to the dependence of Joule heating on electric field and current density and to the dependence of resistivity on temperature. Various discretization techniques have been proposed in literature for the numerical solution of such problems. For AC excitation currents, even after reducing numerical complexity by introducing the time-harmonic electromagnetic field approximation, the solution to induction heating problem is computationally demanding since nonlinear transient time simulations are required [1], [2].

In this paper an approach based on a novel nonlinear Model Order Reduction technique extending [3] and [4], for the numerical solution of induction heating problems, is proposed. Firstly the nonlinear equations of the problem are rewritten in an equivalent way in which only quadratic nonlinearities occur. Secondly, by projecting these equivalent equations in such a way to preserve their nonlinear structure, a reduced nonlinear model, with a few Degrees of Freedom (DoF), is constructed. The projection space is determined computing a few of the first kernels in Volterra series expansion of the solution to the induction heating problem. Such kernels are numerically estimated in an efficient way by solving a succession of uncoupled linear eddy-current and heat diffusion problems, which can be spatially discretized by any numerical method such as the Finite Element Method (FEM). Lastly, the reduced nonlinear model equations are numerically solved in place of the induction heating problem, at negligible computational cost. Using the DoF of the reduced nonlinear model, the whole space-time distribution of the coupled electromagnetic and thermal fields are obtained.

A numerical investigation of a sample numerical application shows that the proposed approach allows to approximate the solution to the discretized induction heating problem with high accuracy in a computational time about one order of magnitude less than that of a standard solution approach [1].

II. Formulation by Quadratic Equations

The eddy current problem equations in the spacial region Ω are written in terms of the magnetic potential vector $\mathbf{a}(\mathbf{r},t)$ as

$$\nabla \times (\nu(\mathbf{r}) \nabla \times \mathbf{a}(\mathbf{r},t)) = \mathbf{j}(\mathbf{r},t) + \mathbf{j}_s(\mathbf{r},t), \tag{1}$$

in which $\nu(\mathbf{r})$ is the reluctivity, $\mathbf{j}(\mathbf{r},t)$ is the eddy current density, and $\mathbf{j}_s(\mathbf{r},t)$ is the source current density, $\mathbf{r}$ being the position vector and t the time instant. In order to get a quadratic nonlinear equation, Ohm's law within the conductive region Ω_c is written in the unusual form

$$-\frac{\partial \mathbf{a}}{\partial t}(\mathbf{r},t) = (\rho(\mathbf{r}) + \delta(\mathbf{r})\vartheta(\mathbf{r},t))\, \mathbf{j}(\mathbf{r},t), \tag{2}$$

in which $\rho(\mathbf{r})$ is the resistivity at room temperature, while $\delta(\mathbf{r})$ is temperature coefficient. The heat diffusion equation in Ω is

$$c(\mathbf{r})\frac{\partial \vartheta}{\partial t}(\mathbf{r},t) + \nabla \cdot (-k(\mathbf{r})\nabla \vartheta(\mathbf{r},t)) = -\frac{\partial \mathbf{a}}{\partial t}(\mathbf{r},t) \cdot \mathbf{j}(\mathbf{r},t), \tag{3}$$

in which $c(\mathbf{r})$ is the volumetric heat capacity and $k(\mathbf{r})$ is the thermal conductivity. For the sake of simplicity, homogeneous conditions are assumed on the boundary $\partial\Omega$, i.e. $\mathbf{a}(\mathbf{r},t) \times \mathbf{n}(\mathbf{r}) = \mathbf{0}$ and $\vartheta(\mathbf{r},t) = 0$, where $\mathbf{n}(\mathbf{r})$ is the outward unit normal.

As common in induction heating applications, the source current density can be assumed in the form

$$\mathbf{j}_s(\mathbf{r},t) = \mathbf{f}(\mathbf{r}) I(t), \tag{4}$$

where $\mathbf{f}(\mathbf{r})$ is spatial distribution of source current density and $I(t)$ is the source current.

III. Volterra series expansion

As it is well known, for a nonlinear dynamic system, under mild regularity hypothesis, any output variable admits a Volterra series expansion, both in the time and complex frequency domains.

In the proposed approach, a few of the first kernels in Volterra series expansions of variables $\mathbf{a}(\mathbf{r},t)$, $\mathbf{j}(\mathbf{r},t)$, $\vartheta(\mathbf{r},t)$ are computed for proper choices of the complex frequency variables. As a consequence of well-known properties of Volterra series [5], this computation is obtained by solving a succession of uncoupled *linear* eddy current problems and of heat diffusion problems, in the complex frequency domain, for chosen values of the frequency variable. The solutions to these problems can be approximated by any proper discretization method, such as FEM. The details will be given in the full paper. By orthonormalizing the computed kernels of Volterra series expansion of $\mathbf{a}(\mathbf{r},t)$, $\mathbf{j}(\mathbf{r},t)$, $\vartheta(\mathbf{r},t)$, bases $\mathbf{a}_k(\mathbf{r})$, $\mathbf{j}_k(\mathbf{r})$, and $\vartheta_k(\mathbf{r})$ are defined. Coupled electromagnetic and thermal fields are thus approximated as

$$\mathbf{a}(\mathbf{r},t)=\sum_{k=1}^{m_a}\mathbf{a}_k(\mathbf{r})\,\hat{a}_k(t),\quad \mathbf{j}(\mathbf{r},t)=\sum_{k=1}^{m_j}\mathbf{j}_k(\mathbf{r})\,\hat{j}_k(t), \tag{5}$$

$$\vartheta(\mathbf{r},t)=\sum_{k=1}^{m_a}\vartheta_k(\mathbf{r})\,\hat{\vartheta}_k(t). \tag{6}$$

The DoF $\hat{a}_k(t)$, $\hat{j}_k(t)$ and $\vartheta_k(t)$ are determined by solving the reduced nonlinear model equations defined hereinafter.

IV. Structure Preserving Reduced Model

The reduced nonlinear model equations are derived by separately projecting (1)-(3) onto the spaces defined by (5)-(6). Thus, taking the dot product of both members of (1) with $\mathbf{a}_k(\mathbf{r})$ and integrating over Ω it results in the ordinary differential equations (ODEs)

$$\sum_{j=1}^{m_a}\hat{n}_{ij}\frac{d\hat{a}_j}{dt}(t)=\sum_{j=1}^{m_j}\hat{e}_{ij}\hat{j}_j(t)+\hat{f}_i I(t), \tag{7}$$

taking the dot product of both members of (2) with $\mathbf{j}_i(\mathbf{r})$ and integrating over Ω_c it results in the ODEs

$$-\sum_{j=1}^{m_j}\hat{e}_{ji}\frac{d\hat{a}_j}{dt}=\sum_{j=1}^{m_j}\left(\hat{r}_{ij}+\sum_{k=1}^{m_\vartheta}\hat{m}_{ijk}\,\hat{\vartheta}_k(t)\right)\hat{j}_j(t), \tag{8}$$

multiplying both members of (3) by $\vartheta_i(\mathbf{r})$ and integrating over Ω it results in the ODEs

$$\sum_{j=1}^{m_\vartheta}\left(\hat{c}_{ij}\frac{d\hat{\vartheta}_j}{dt}(t)+\hat{k}_{ij}\hat{\vartheta}_j(t)\right)=-\sum_{j=1}^{m_a}\sum_{k=1}^{m_j}\hat{p}_{ijk}\frac{d\hat{a}_j}{dt}\hat{j}_k(t). \tag{9}$$

By solving the reduced model equations (7)-(9) for its DoF and using (5)-(6), the coupled electromagnetic and thermal fields are reconstructed in the whole domain Ω and for any time instant t.

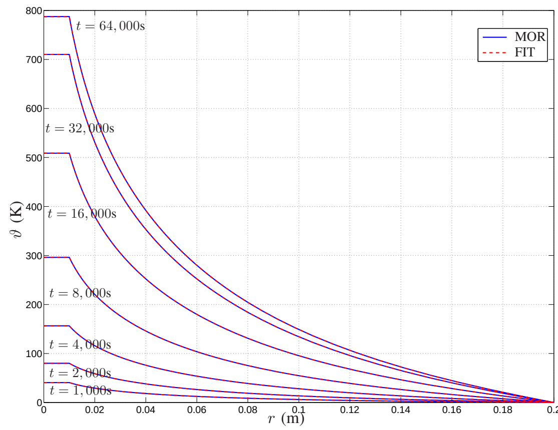

Fig. 1. Temperature rise ϑ provided by FIT and reconstructed from the compact model, along the r axis at various time instants t.

V. Numerical Results

Induction heating of a cylindrical billet having 1 cm radius and 1 cm length made of copper is analyzed. An excitation is introduced through a current-driven coil of 5 cm inner radius, 2 cm thickness, 2 cm length. An AC excitation at 500Hz is considered so that a time-harmonic eddy current problem is considered.

A two-dimensional axisymmetrical model which exploits the cylindrical symmetry of the problem is discretized by the Finite Integration Technique (FIT) using about $40,000$ unknowns. A reduced nonlinear model, having 45 DoFs, is attained in less than 10s on a 2.3 Ghz Intel Core i7, by computing the first 5 kernels of Volterra series expansions evaluated at 10 values of complex frequency on the real axis.

The numerical solution to the reduced nonlinear model equations, evaluated for an induction heating time of $70,000$s, requires less than 1s, whereas with the numerical solution of the full problem, requires about 15 min. Using the DoFs of the reduced nonlinear model, the whole space-time distribution of the coupled electromagnetic and thermal fields are accurately approximated at each spatial point and time instant, with a relative error of about 0.1% in the maximum norm. Such a high accuracy can be appreciated in Fig. 1, in which the temperature rise ϑ provided by FIT, along the radial axis r, is compared at various time instants to the temperature rise reconstructed using the reduced nonlinear order, with an error smaller than 0.1K.

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